The Plain Arithmetic of Social Security

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1 Introduction

The object of this article is to encourage skepticism over an argument that is often made by those who advocate privatization of Social Security. The argument in question is that the historical rate of return on individuals' "investments" in Social Security is poor compared with the historical return on stocks, and therefore people could have done better for themselves by buying assets of their own choosing, had they not been obliged to contribute to a government-mandated scheme. For a hearty polemic to this effect see, e.g., Reynolds (2005); for a more detailed argument see Garrett and Rhine (2005).

It is not particularly easy to determine the rate of return on stocks over a long sweep of history, but the figure arrived at by Siegel (1994)—namely, an annual real return of about 7 percent—seems to be widely accepted. This figure has provoked some debate recently, with Zweig (2009) expressing skepticism over the 19th century data used by Siegel, and Arnott (2009) arguing that stocks have not outrun bonds over the long haul. For my purposes here it is not necessary to arrive at the "correct" figure for the long-run return on stocks; it is sufficient to note what many commentators on Social Security *believe* that return to be, and 7 percent will serve.

Nonetheless, it may worth taking a quick look at some data. Figure 1 shows the moving per-decade real return on the S&P 500, based on monthly total returns from January 1970 to July 2009. The plot represents the annualized real gain that would have been achieved by buying into the S&P 500 ten years prior to month t, reinvesting dividends, then selling at t.[1] While anyone who bought into the S&P 500 in the early to mid 1970s or the late 1990s would have been out of luck, we see that the per-decade return was over 7 percent for most of the period between 1986 and 2005. The sample mean of the monthly observations is 7.3 percent. Of course, a decade is too short a period to be very relevant to the issue of saving for retirement, and the variance is high, but it seems that Siegel's 7 percent is not out of the ballpark for this sample period.

But is it reasonable to suppose that the bulk of the working population could have made returns such as this, had they not been fettered by Social Security? I will argue that the answer is No.

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[1] For the period 1970–1988 I used the series `TRSP500` from the St Louis Fed; for 1988–2009 I constructed a comparable series using one-month total returns data from the Standard and Poor's website, `http://www2.standardandpoors.com/spf/xls/index/MONTHLY.xls`. Let y_t denote the level of the S&P index, augmented to represent monthly reinvestment of dividends and deflated by the CPI: the series graphed is then $(\log y_t - \log y_{t-120})/10$.

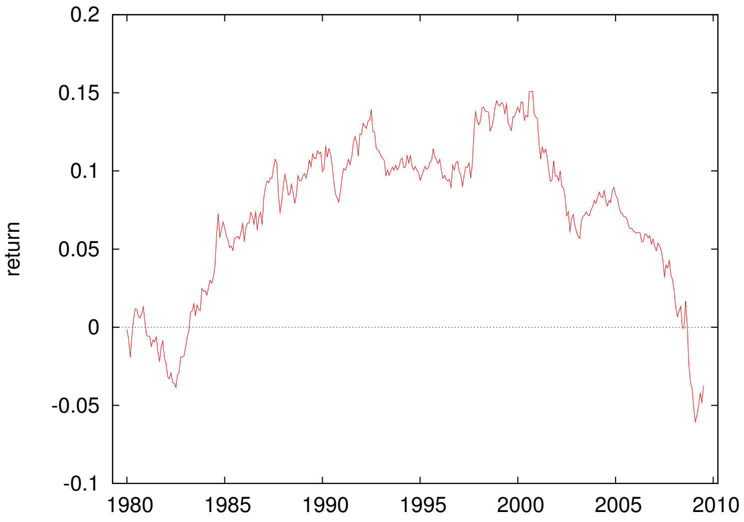

Figure 1: Real per-decade returns on the S&P 500, 1980–2003

Some caveats

To avoid disappointing the reader, let me make two points clear at the outset.

First, this note has nothing to say directly about the big issue facing Social Security over the next few decades, namely, the effects flowing from the aging of the baby-boom generation. I confine myself here to the arithmetic of Social Security in the steady state (which I will define below). But I believe that the argument presented here is relevant in this sense: getting clear on the laws governing saving for retirement in the steady state would seem to be a prerequisite for getting clear on the more complex question of behavior out of steady state.

Second, I am not concerned with the details of the US Social Security system; rather, I'm treating it as an exemplar of the general class of "pay as you go" systems, in which contributions paid in by today's workforce flow out immediately to today's retirees. In fact, the US system has departed from this scheme to some extent since 1983, when Social Security taxes were raised in an effort to build up a fund against the anticipated fall in the ratio of workers to retirees in the 21st century. But that's precisely the sort of non-steady state issue from which I'm abstracting. So, despite my title, it might be less confusing if I choose a different name for the scheme whose laws I'm trying to discern; let's call it "Retirement."

Retirement

"Retirement," then, is a scheme whereby workers pay a certain percentage of their income into a central "kitty," from which it is disbursed to retirees. Suppose that in year t there are n_1 workers and n_2 retirees. Let per capita income from work be Y_t; let workers pay a fraction τ of their incomes into Retirement, and let retirees draw a fraction λ of per capita working income from Retirement. The situation is then as shown in Figure 2.

The "pay as you go" balance condition is that $n_1 \tau Y_t = n_2 \lambda Y_t$, which we may write as

$$\lambda = \tau \bar{w} \quad \text{or} \quad \tau = \frac{\lambda}{\bar{w}} \tag{1}$$

Figure 2: Pay as you go

where $\bar{w}$ is shorthand for n_1/n_2, the ratio of workers to retirees.

The key aspect of the steady state, for such a system, is that $\bar{w}$ remains constant over time. We will also assume that λ remains constant (and therefore so does τ). The second point is not so much a separate stipulation as a consequence of the first. We require that the ratio of workers' to retirees' per capita income should have no trend (otherwise the income of either workers or retirees would go to zero in the limit). And the simplest case of "no trend" is constancy, represented by constant λ.

These conditions are consistent with steady growth of per capita income, and also with a steady rate of change of population. And they do not prevent us from considering variations in $\bar{w}$ or λ so long as the thought experiment takes the form of a comparison of steady states. In fact comparison of steady-state positions is the name of the game in the following.

My central claim is that there are strict bounds on the "rate of return" that can be gained from Retirement, and that these mostly have to do with (a) the underlying growth rate of per capita income and (b) demography. It may seem at first glance that this claim, even if true, doesn't address the view that I am disputing—that retirees could do better if they purchased assets of their own choosing—but that is wrong. While Retirement might most naturally be interpreted as an idealized version of Social Security, in fact it is much more general: given only the steady state conditions, *any* system whereby today's workers put money into Retirement, and today's retirees collect from Retirement and spend, must mimic an explicitly pay-as-you-go system. This is perhaps easiest to see if we imagine a system in which stocks pay no dividend, the entire return taking the form of capital gains. Today's workers buy stocks; today's retirees spend out of sales of stock. The balance condition is that sales and purchases match, the same as pay-as-you-go.[2]

The arithmetic of Retirement is examined below by means of simulation.

2 Parameters of the simulation

Some features of the simulation remain constant across all variants, while others are varied to provide comparative results.

The basic invariant is this: people are assumed to start paying into Retirement at age 25, and to continue paying a constant fraction τ of their income until a definite age—65 in the base case—at which point they retire. Thereafter they draw from Retirement until age 85, at which point they die (or at least, stop receiving payments).

A further invariant is that real income per capita is assumed to grow at 2 percent per year. One may think of the economy as being in the Solow steady state, with the growth of per capita income governed by the long-run rate of technical progress. People have a right to an income from Retirement that is a given fraction, λ, of income from work. That is, everyone's

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[2] This is arguably an over-simplification—both for Retirement as outlined above and for a system based on purchase and sale of stock—if the steady-state population growth rate is positive. We return to this point at the end of section 4 below.

real income grows at 2 percent, except that there is a discontinuity upon retirement whereby income drops to λ times current working income.[3]

As stated above, there is a simple linear relationship between the "generosity" of Retirement, as measured by λ, and the contribution rate τ, namely $\lambda = \tau\bar{w}$. For example, suppose there are three workers per retiree ($\bar{w} = 3$). If the contribution rate is 0.2 then retirees will receive 0.6 of a working income. Or if $\lambda = 0.5$, the required contribution rate is $0.5/3 = 0.167$.

The number of workers per retiree depends on three main factors: the rate of population growth, the shape of the survival curve (defined below), and the retirement age. Section 3 considers the case of zero population growth and section 4 the case of population growth at the rate experienced in the US over the past few decades. In each section we consider two variants of the survival curve and also variant settings of the retirement age. It will be useful first to discuss the survival curve.

The survival curve

By the survival curve we mean the function that maps from age to the number of people, from an initial birth cohort of a given size, who are still living at that age. As stated in section 2, we assume that people pay into Retirement from age 25, so what matters in the present context is the curve from age 25 on. (If you die before age 25, you're not in this model.) Figure 3 shows the US Life Table for 2004 (with age given at five-year intervals) and plots the curve from age 25 along with a cubic in age that fits very well, namely

$$\widehat{\text{survivors}} = \underset{(2969.3)}{124481} - \underset{(183.62)}{2050.23}\,\text{age} + \underset{(3.5248)}{52.6072}\,\text{age}^2 - \underset{(0.021262)}{0.475005}\,\text{age}^3 \qquad (2)$$

(standard errors in parentheses, $R^2 = 0.9997$).[4]

A steeper survival curve will be associated, ceteris paribus, with a greater number of workers per retiree, and hence, via (1), with a lower contribution rate, τ, for any given value of the "generosity ratio", λ.

3 Zero population growth

We start by considering the case of zero population growth. Even if this were unambiguously "unrealistic" it would still be worthwhile to consider as a baseline, in order to isolate the effect of population growth. In fact, of course, it *is* fairly realistic in relation to many advanced economies.

Flat survival curve

We have set out data on the US survival curve above, but as an initial benchmark we consider the behavior of the system under a flat survival curve. By this we mean that everyone who is

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[3] Note that the 2 percent growth figure is a little over-generous in relation to the performance of the US economy over the last 60 years. Using quarterly data on real GDP and population (series `GDPC96` and `CNP16OV` from the St Louis Fed), the regression of the log of GDP per capita on time gives an annualized growth rate of 1.84 percent for the period 1948Q1 to 2009Q2.

[4] A quadratic fit is quite good ($R^2 = 0.983$) but has the unsatisfactory implication that the survival curve slopes upward over part of the age range. A quartic fit is even better (higher adjusted R^2), but seems like overkill.

Age	Survivors
0	100000
1	99320
5	99202
10	99129
15	99036
20	98709
25	98246
30	97776
35	97250
40	96517
45	95406
50	93735
55	91357
60	88038
65	83114
70	76191
75	66605
80	53925
85	38329

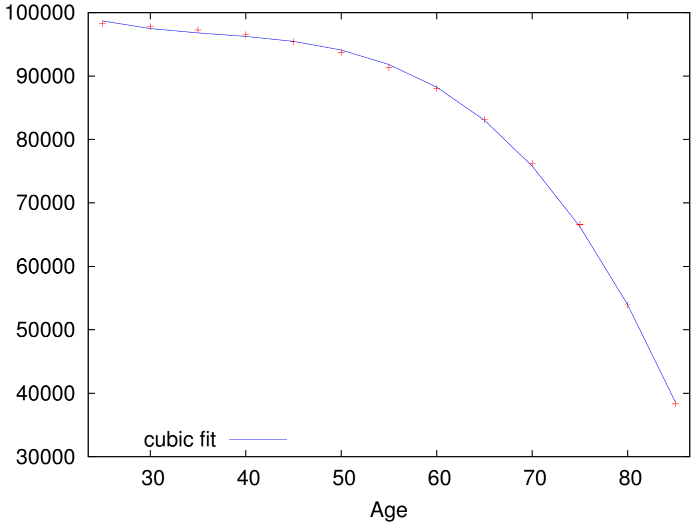

Figure 3: US survival curve, table at left from Arias (2007)

alive at age 25 lives to age 85—that is, everyone gets to enjoy the full benefits of Retirement. This is clearly unrealistic but it enables us to see just how much difference mortality makes.

Calculation of the number of workers per retiree is particularly easy in this case: it is just the ratio of the numbers of years of working life (the retirement age minus 25) to the number of years of retirement (85 minus the retirement age).

Let A denote age in years and $A^\star$ denote the retirement age; let Y denote real income and g its growth rate. The simulation algorithm is then as follows:

1. Parameters: $A^\star = 65$, $\lambda = 0.5$, $g = 0.02$.
2. Flat survival curve: $\bar{w} = (A^\star - 25)/(85 - A^\star) = 40/20 = 2$; $\tau = \lambda/\bar{w} = 0.25$.
3. $A = 25$, $Y = 10{,}000$.
4. If $A < A^\star$ then record Retirement contribution of τY, else record Retirement income of λY.
5. $A \leftarrow A + 1$. If $A < 85$, $Y \leftarrow (1 + g)Y$ and go to step 4, else stop.

Note that step 4 is last repeated for age $A = 84$; this is the 85th year, since one's Nth birthday occurs at the end of year N, or in other words someone who is of age N is in their $N + 1$ year.

Detailed output from all the simulations is shown in the Appendix; the one we are considering here is Simulation 1. The first column shows a representative individual's age in years and the second shows income from work, arbitrarily initialized to 10,000. The third column shows the contribution to Retirement or disbursement from Retirement, and the fourth shows the straight cumulated balance (at a discount rate of zero).

Once the main algorithm terminates we calculate the Internal Rate of Return (IRR) of the recorded stream of contributions (negative) and disbursements (positive). This is the discount rate at which the net present value of the stream is zero.

In Simulation 1 the IRR is 0.02. It should come as no surprise to see that, with zero population growth and a flat survival curve, the IRR equals the assumed growth rate of per capita income. What may be slightly less obvious at first glance is that this result is truly "pinned in place" by the basic demographic assumptions.

First question: what happens if we alter λ, making the system more or less "generous" to retirees? Answer: there is no change in the rate of return. We ran a simulation with $\lambda = 0.6$ to verify this, but since the reason for the neutrality of λ is easily grasped *a priori* this run is not reported in the Appendix. This is a pure "pay as you go" system, but from the individual's point of view the system provides a means of transferring income forward in time at a rate of return equal to the growth rate. The *amount* of income transferred in this way (governed by λ) makes no difference to the rate of return—so long as we adjust τ correspondingly as in equation (1). This result applies regardless of the demographic assumptions made.[5]

Second question: what happens if we alter the retirement age? Simulation 2 shows the results for $A^\star = 70$: the IRR remains at 0.02. Unlike variation in λ, however, this neutrality result depends on the assumption of a flat survival curve. Raising the retirement age at given λ (a) raises the number of workers per retiree and so (b) lowers the contribution rate, τ, while (c) reducing the total disbursement per retiree by reducing the length of retirement. Under the flat survival curve, but not in general, these effects cancel and leave the rate of return unaffected.

With a realistic survival curve

If some retirees die before 85 and hence fail to take full advantage of their entitlement—but we maintain the "pay as you go" balance condition—then the survivors will garner a higher rate of return. In Simulation 3 we assume a retirement age of 65 along with $\lambda = 0.5$ and $g = 0.02$ (as in Simulation 1), but instead of a flat survival curve we use equation (2) to compute the relative numbers of people alive at each year of age, and hence the number of workers per retiree.

To be precise, the algorithm used to compute $\bar{w}$ is as follows, where $f(\cdot)$ denotes the cubic given by (2):

1. $n_1 = n_2 = 0$; $A = 25$.
2. If $A < A^\star$, $n_1 \leftarrow n_1 + f(A)$, otherwise $n_2 \leftarrow n_2 + f(A)$.
3. $A \leftarrow A + 1$. If $A < 85$ then go to step 2, else $\bar{w} = n_1/n_2$ and stop.

Table 1 summarizes the comparison: under the realistic curve the number of workers per retiree is greater and the required contribution rate is therefore lower. The IRR rises from 2 percent to 3.21 percent. But note that the latter figure applies only to those who survive to age 85; it would be misleading to present it as "the" rate of return. Those who live to 85 constitute $\frac{38329}{98246}$ or 39 percent of the cohort as of age 25 (see Figure 3). Those who live to a lesser age get a lesser rate of return, and in fact we can see from the "Balance" column in Simulation 3 (Appendix) that those who die before age 73 make a negative return. (The "Balance" figures are undiscounted, so, given the timing of outgoings and receipts, a negative balance means a sub-zero IRR.)

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[5] Just to be sure, we ran simulations with $\lambda = 0.6$, as well as the baseline $\lambda = 0.5$, for all combinations of demographic assumptions. But since this simply confirmed the argument given in the text these are not reported.

	Survival curve:	
	flat	realistic
Workers per retiree, $\bar{w}$	2.00	2.87
Contribution rate, τ	0.250	0.174
(Maximal) rate of return	0.0200	0.0321

Table 1: Comparison of Simulations 1 and 3: survival curve effects for $\lambda = 0.5$, $A^{\star} = 65$, zero population growth

The relatively high maximal return under the realistic survival curve is of interest in its own right, but the "fair" rate of return is arguably still 2 percent as in Simulation 1. In an appropriate sense this is the average across the lucky and less lucky individuals.

In Simulation 4 we repeat the experiment of raising the retirement age. This time the change is non-neutral: raising the retirement age from 65 to 70 raises the maximal rate of return from 3.21 percent to 3.4 percent. In general, with the declining survival curve, raising the retirement age raises the ratio of workers to retirees, hence lowering τ and raising the maximal rate of return. A broader view of this effect is shown in Figure 4, which is based on a series of simulations on the pattern of Simulation 4 but with retirement age running from 60 to 84. At a retirement age of 84 the maximal return reaches 4.22 percent. We should emphasize that returns of this magnitude are a "cheat": *some* people could get over 4 percent with a high enough retirement age, but only on condition that the majority get a negative rate.

Retirement	Maximal IRR
60	0.03059
65	0.03215
70	0.03403
75	0.03635
80	0.03924
81	0.03991
82	0.04061
83	0.04136
84	0.04215

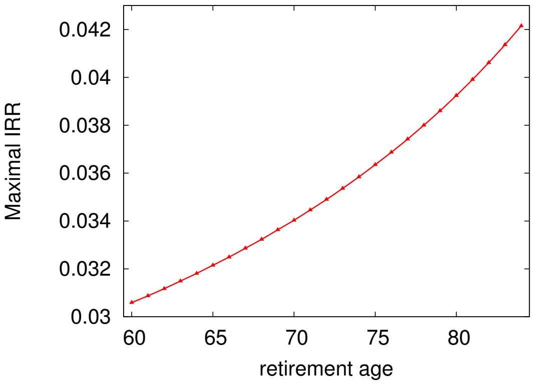

Figure 4: Maximal rate of return versus retirement age, given zero population growth and realistic survival curve

What, in the end, should we make of the simulation with the realistic survival curve? I have argued that the maximal return is a misleading figure, yet it's true that a flat survival curve is unrealistic. My reading is that while the "fair" rate of return remains at 2 percent, Simulation 3 is correct in telling us that we don't need $\tau = 0.25$ to achieve this. Given the sad facts of mortality, we can get by with $\tau = 0.174$. Assuming, that is, that Retirement benefits are in no way transferable. If benefits are transferred, to some extent, to surviving family members when the primary beneficiary dies (as is the case in the US Social Security system), the maximal rates computed above are not applicable: the effective value of $\bar{w}$ is less than we assumed, and τ higher.

It is perhaps worth noting that it's difficult to come up with a good definition of the "fair" or average rate of return *other than* that which emerges from the counterfactual assumption of a flat survival curve. One might think of calculating, in relation to Simulation 3, the probability-weighted mean of the rates of return accruing to the sets of people who live to age 26, 27, ..., 85, but this won't work since anyone who dies before retirement age gets an IRR of $-\infty$. Another possibility would be to compute the rate of return—again in relation to Simulation 3—for those who just reach the life expectancy implied by the survival function $f(\cdot)$. Life expectancy is somewhat stylized under our assumptions: it is conditional on reaching age 25 and we assume life ends at 85 for everyone. It can be calculated using

$$E(\text{life}) = \sum_{A=25}^{85} P(\text{life} = A) \times A$$

where

$$\begin{aligned} P(\text{life} = A) &= P\left((\text{survive to } A) \wedge \neg(\text{survive to } A+1)\right) \\ &= \frac{f(A)}{f(25)} \times \begin{cases} 1 - f(A+1)/f(A) & \text{if } A < 85, \\ 1 & \text{if } A = 85 \end{cases} \end{aligned}$$

Based on the $f(\cdot)$ reported above, this yields a life expectancy of 75.82. Calculating the IRR when the disbursement stream is truncated at ages ranging from 69 to 84 (on the assumptions stated above, with retirement at 65) we get the curve shown in Figure 5. We see that the return gets to 2 percent by about 79; those who die at 76 get a little over 1 percent. So the return at life expectancy falls short of what I am claiming is the "fair" 2 percent.[6]

Final age	IRR
69	−0.04977
70	−0.03323
75	0.00611
76	0.01040
77	0.01410
78	0.01734
79	0.02020
80	0.02275
84	0.03061

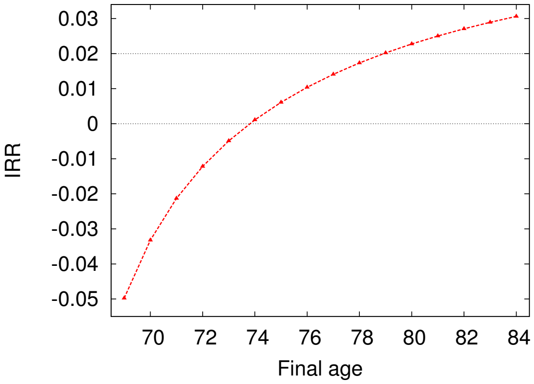

Figure 5: IRR versus final age, retirement age 65

4 Positive population growth

In this section we examine the effects of population growth at something like the rate experienced in recent decades in the US. Specifically, we use the rate of growth of the civilian non-institutional population from January 1980 to August 2009, as gauged by a logarithmic regression on a constant and time trend.[7] This gives an annualized growth rate of $n = 0.0118$,

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[6] Given the treatment of age as a discrete variable one would not expect exact agreement, but the size of the discrepancy would seem to indicate that the two magnitudes are not conceptually equal.

[7] The population series is `CNP160V` from the BLS via the Federal Reserve Bank of St Louis.

or a little over 1 percent. Immigration has contributed to the growth of US population over this period, but we assume a idealized version in which each year's birth cohort is $(1 + n)$ times the size of the previous cohort.

The basic effect of steady population growth seems clear: since each birth cohort is larger than the preceding one this will raise the number of workers per retiree and lower the contribution rate required for any given λ, leading to a higher return from Retirement.

With a flat survival curve

We first replicate the analysis of section 3, assuming that everyone alive at 25 survives to age 85. In this case the algorithm to compute $\bar{w}$ is:

1. $n_1 = n_2 = 0$; $A = 25$; $C = 10{,}000$.
2. If $A < A^\star$, $n_1 \leftarrow n_1 + C$, otherwise $n_2 \leftarrow n_2 + C$.
3. $A \leftarrow A + 1$. If $A < 85$ then $C \leftarrow C(1 + g)^{-1}$ and go to step 2, else $\bar{w} = n_1/n_2$ and stop.

C represents the size of a birth cohort; cohorts at greater ages were born earlier and so are smaller, since $g > 0$.

In Simulation 5 we set $\lambda = 0.5$ and the retirement age is 65. Table 2 shows a summary comparison: we see that the rate of return is now 3.2 percent, as opposed to 2 percent with zero population growth.

	Population growth:	
	zero	0.0118
Workers per retiree, $\bar{w}$	2.00	2.86
Contribution rate, τ	0.250	0.175
Rate of return	0.0200	0.0320

Table 2: Comparison of Simulations 1 and 5: population growth effects for $\lambda = 0.5$, $A^\star = 65$

Simulation 6 puts the retirement age at 70: the rate of return remains at 3.2 percent. As with the comparison of Simulations 1 and 2, raising the retirement age makes no difference to the rate of return given the flat survival curve.

With the realistic survival curve

Our final set of simulations combines positive population growth with the realistic survival curve. The algorithm for $\bar{w}$ is now:

1. $n_1 = n_2 = 0$; $A = 25$; $C = 1$.
2. If $A < A^\star$, $n_1 \leftarrow n_1 + C \cdot f(A)$, otherwise $n_2 \leftarrow n_2 + C \cdot f(A)$.
3. $A \leftarrow A + 1$. If $A < 85$ then $C \leftarrow C(1 + g)^{-1}$ and go to step 2, else $\bar{w} = n_1/n_2$ and stop.

The retirement age is 65 in Simulation 7 and 70 in Simulation 8. Table 3 gives a summary. These runs give us the highest maximal rates of return, but as before we emphasize that they are not "fair" rates, in that they are inherently unavailable to the majority of participants.

	Retirement age:	
	65	70
Workers per retiree, $\bar{w}$	4.08	6.58
Contribution rate, τ	0.123	0.076
Maximal rate of return	0.0436	0.0455

Table 3: Summary of Simulations 7 and 8: retirement age effects

Is the population growth effect real?

We seem to have shown that positive population growth boosts the rate of return to Retirement, by raising the number of workers per retiree, but arguably we have missed something from the larger macroeconomic picture.[8]

Let us suppose for a moment that Retirement is merged into a larger agency—call it Planning—which is charged, not only with ensuring a reasonable income for retirees, but also with ensuring that the capital stock grows *pari passu* with the growth of the labor force. The Retirement contribution τ is then subsumed within a rate of contribution to Planning that allows for a suitable rate of net investment. If an additional deduction were *not* made for expansion of capital stock, the assumed per capita growth rate of 2 percent could not be sustained, undermining the basis of our simulations.[9]

Maddison (1995) reckoned the ratio of non-residential capital stock to GDP in the US to be about 2.4 as of 1992. Take this as a ballpark figure. If growth of population at 1.18 percent requires growth of capital stock at the same rate, this implies net investment of about $1.18 \times 2.4 = 2.8$ percent of GDP. If labor income is around $\frac{2}{3}$ of GDP, that's about 4.2 percent of labor income. We'd get a higher figure here if we used the estimate of the US capital–output ratio given by Mankiw, Romer and Weil (1992), namely 3.

Now recall that under the realistic survival curve, and with retirement age 65, we had $\tau = 0.174$ for the case of zero population growth (Simulation 3) while we obtained $\tau = 0.123$ when assuming 1.18 percent population growth (Simulation 7), for a difference of 0.051. It now looks as if most if not all of this differential would in fact be eaten up by the additional contribution to Planning. But then the fair return to the contribution to Planning would be well approximated by the 2 percent that we calculated initially.

5 Conclusion

We have made the argument that the fair return to Retirement—which can be considered a simplified and stylized version of the Social Security system, but whose basic constraints apply to any sustainable system of providing retirement income—is tethered to the long-run growth of per capita income. In a Solow steady state this is equal to the rate of technical progress; as an empirical matter is it slightly less than 2 percent in recent US history.

We have considered the ways in which demographic factors can act to raise the return above the 2 percent level, but have also made the case that the "demographic boost" is not

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[8] The fact that the Solow growth model associates higher population growth with lower steady-state per capita income might be a warning signal.

[9] The counterpart to this requirement in a system based on purchases and sales of stock is that purchases by workers must exceed sales by retirees, so as to take up the new issues associated with expansion of the capital stock.

all that it seems. A declining survival curve raises returns for those who live longest, but at the expense of those who die earlier. Steady population growth raises the rate of return on Retirement itself, but one must consider the offsetting requirement that a portion of income be devoted to expansion of capital stock. In any case, the maximum rate of return we were able to produce (for the longest-lived retirees only, with a retirement age of 70, and ignoring the capital stock issue) was about 4.5 percent.

In this light, how could it be possible for the bulk of the population to make returns on the order of 7 percent, simply by switching their retirement contributions into acquisition of stocks? I don't see anyone arguing that this policy would raise per capita growth to 7 percent. The notion seems to be rooted in the fallacy of composition. We all know about this fallacy but we're not always sufficiently alert to it. We smile when Garrison Keillor tells us that in Lake Wobegon "all the children are above average". We should be equally amused when people tell us that we could all earn an above-average rate of return if we got out of Social Security and into stocks.

How can the long-run average return on stocks be 7 percent (if it is) in a 2 percent economy? That's an interesting question. I won't attempt to resolve it here, but simply make a couple of brief observations. First, obviously some people can make above average returns on their saving provided others are getting below average; and if the owners of stock are not too numerous—or, at any rate, if the distribution of stock-ownership is sufficiently skewed—then stock returns could be substantially above the growth rate without violating the laws of arithmetic. But secondly there's the question, just how real are the returns calculated by Siegel and others? For sure, they are "real" in the sense that they're deflated by a price index, but the proof of the pudding is in the eating. A relevant difference between the historical business of buying stocks, on the one hand, and contributing to Social Security, on the other, is that in the first case wealth is heritable, and at the top of the wealth distribution is often passed on in large quantities, while Social Security "wealth" is consumed in retirement. What would happen if stock holders attempted to consume their wealth within their lifetimes? In a sense we would then find out what is it really worth.

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Appendix: simulation output

A. Zero population growth plus flat survival curve

Simulation 1: $A^\star = 65$

```
per capita growth rate = 0.02
population growth = 0
flat survival curve
retirement at 65, worklife 40 yrs,
 retirement 20 yrs
retirement income: lambda = 0.5
workers per retiree = 2
implied tau = 0.25

Age    Income  Retirement   Balance
 25   10000.0     -2500.0   -2500.0
 26   10200.0     -2550.0   -5050.0
 27   10404.0     -2601.0   -7651.0
 28   10612.1     -2653.0  -10304.0
 29   10824.3     -2706.1  -13010.1
 30   11040.8     -2760.2  -15770.3
 31   11261.6     -2815.4  -18585.7
 32   11486.9     -2871.7  -21457.4
 33   11716.6     -2929.1  -24386.6
 34   11950.9     -2987.7  -27374.3
 35   12189.9     -3047.5  -30421.8
 36   12433.7     -3108.4  -33530.2
 37   12682.4     -3170.6  -36700.8
 38   12936.1     -3234.0  -39934.8
 39   13194.8     -3298.7  -43233.5
 40   13458.7     -3364.7  -46598.2
 41   13727.9     -3432.0  -50030.2
 42   14002.4     -3500.6  -53530.8
 43   14282.5     -3570.6  -57101.4
 44   14568.1     -3642.0  -60743.4
 45   14859.5     -3714.9  -64458.3
 46   15156.7     -3789.2  -68247.5
 47   15459.8     -3864.9  -72112.4
 48   15769.0     -3942.2  -76054.7
 49   16084.4     -4021.1  -80075.7
 50   16406.1     -4101.5  -84177.3
 51   16734.2     -4183.5  -88360.8
 52   17068.9     -4267.2  -92628.0
 53   17410.2     -4352.6  -96980.6
 54   17758.4     -4439.6 -101420.2
 55   18113.6     -4528.4 -105948.6
 56   18475.9     -4619.0 -110567.6
 57   18845.4     -4711.4 -115278.9
 58   19222.3     -4805.6 -120084.5
 59   19606.8     -4901.7 -124986.2
 60   19998.9     -4999.7 -129985.9
 61   20398.9     -5099.7 -135085.6
 62   20806.9     -5201.7 -140287.3
 63   21223.0     -5305.7 -145593.1
 64   21647.4     -5411.9 -151005.0

 65              +11040.2 -139964.8
 66              +11261.0 -128703.8
 67              +11486.2 -117217.5
 68              +11715.9 -105501.6
 69              +11950.3  -93551.3
 70              +12189.3  -81362.1
 71              +12433.1  -68929.0
 72              +12681.7  -56247.3
 73              +12935.4  -43311.9
 74              +13194.1  -30117.9
 75              +13457.9  -16659.9
 76              +13727.1   -2932.8
 77              +14001.6   11068.8
 78              +14281.7   25350.5
 79              +14567.3   39917.8
 80              +14858.7   54776.4
 81              +15155.8   69932.3
 82              +15458.9   85391.2
 83              +15768.1  101159.3
 84              +16083.5  117242.8

IRR = 0.0200
```

Simulation 2: $A^\star = 70$

```
per capita growth rate = 0.02
population growth = 0
flat survival curve
retirement at 70, worklife 45 yrs,
 retirement 15 yrs
retirement income: lambda = 0.5
workers per retiree = 3
implied tau = 0.166667

Age    Income  Retirement   Balance
 25   10000.0     -1666.7   -1666.7
 26   10200.0     -1700.0   -3366.7
 27   10404.0     -1734.0   -5100.7
 28   10612.1     -1768.7   -6869.3
 29   10824.3     -1804.1   -8673.4
 30   11040.8     -1840.1  -10513.5
 31   11261.6     -1876.9  -12390.5
 32   11486.9     -1914.5  -14304.9
 33   11716.6     -1952.8  -16257.7
 34   11950.9     -1991.8  -18249.5
 35   12189.9     -2031.7  -20281.2
 36   12433.7     -2072.3  -22353.5
 37   12682.4     -2113.7  -24467.2
 38   12936.1     -2156.0  -26623.2
 39   13194.8     -2199.1  -28822.4
 40   13458.7     -2243.1  -31065.5
 41   13727.9     -2288.0  -33353.5
 42   14002.4     -2333.7  -35687.2
 43   14282.5     -2380.4  -38067.6
 44   14568.1     -2428.0  -40495.6
 45   14859.5     -2476.6  -42972.2
 46   15156.7     -2526.1  -45498.3
 47   15459.8     -2576.6  -48074.9
 48   15769.0     -2628.2  -50703.1
 49   16084.4     -2680.7  -53383.8
 50   16406.1     -2734.3  -56118.2
 51   16734.2     -2789.0  -58907.2
 52   17068.9     -2844.8  -61752.0
 53   17410.2     -2901.7  -64653.7
 54   17758.4     -2959.7  -67613.5
 55   18113.6     -3018.9  -70632.4
 56   18475.9     -3079.3  -73711.7
 57   18845.4     -3140.9  -76852.6
 58   19222.3     -3203.7  -80056.3
 59   19606.8     -3267.8  -83324.1
 60   19998.9     -3333.1  -86657.3
 61   20398.9     -3399.8  -90057.1
 62   20806.9     -3467.8  -93524.9
 63   21223.0     -3537.2  -97062.1
 64   21647.4     -3607.9 -100670.0
 65   22080.4     -3680.1 -104350.0
 66   22522.0     -3753.7 -108103.7
 67   22972.4     -3828.7 -111932.4
 68   23431.9     -3905.3 -115837.8
 69   23900.5     -3983.4 -119821.2

 70              +12189.3 -107631.9
 71              +12433.1  -95198.9
 72              +12681.7  -82517.1
 73              +12935.4  -69581.8
 74              +13194.1  -56387.7
 75              +13457.9  -42929.8
 76              +13727.1  -29202.7
 77              +14001.6  -15201.0
 78              +14281.7    -919.4
 79              +14567.3   13647.9
 80              +14858.7   28506.6
 81              +15155.8   43662.4
 82              +15458.9   59121.4
 83              +15768.1   74889.5
 84              +16083.5   90973.0

IRR = 0.0200
```

B. Zero population growth plus U.S. survival curve

Simulation 3: $A^\star = 65$

```
per capita growth rate = 0.02
population growth = 0
U.S. survival curve
retirement at 65, worklife 40 yrs,
 retirement 20 yrs
retirement income: lambda = 0.5
workers per retiree = 2.872691983
implied tau = 0.174053

Age    Income  Retirement   Balance
 25   10000.0     -1740.5   -1740.5
 26   10200.0     -1775.3   -3515.9
 27   10404.0     -1810.8   -5326.7
 28   10612.1     -1847.1   -7173.8
 29   10824.3     -1884.0   -9057.8
 30   11040.8     -1921.7  -10979.5
 31   11261.6     -1960.1  -12939.6
 32   11486.9     -1999.3  -14938.9
 33   11716.6     -2039.3  -16978.2
 34   11950.9     -2080.1  -19058.3
 35   12189.9     -2121.7  -21180.0
 36   12433.7     -2164.1  -23344.1
 37   12682.4     -2207.4  -25551.5
 38   12936.1     -2251.6  -27803.1
 39   13194.8     -2296.6  -30099.7
 40   13458.7     -2342.5  -32442.2
 41   13727.9     -2389.4  -34831.6
 42   14002.4     -2437.2  -37268.7
 43   14282.5     -2485.9  -39754.6
 44   14568.1     -2535.6  -42290.2
 45   14859.5     -2586.3  -44876.6
 46   15156.7     -2638.1  -47514.6
 47   15459.8     -2690.8  -50205.5
 48   15769.0     -2744.6  -52950.1
 49   16084.4     -2799.5  -55749.6
 50   16406.1     -2855.5  -58605.1
 51   16734.2     -2912.6  -61517.8
 52   17068.9     -2970.9  -64488.7
 53   17410.2     -3030.3  -67519.0
 54   17758.4     -3090.9  -70609.9
 55   18113.6     -3152.7  -73762.6
 56   18475.9     -3215.8  -76978.4
 57   18845.4     -3280.1  -80258.5
 58   19222.3     -3345.7  -83604.2
 59   19606.8     -3412.6  -87016.8
 60   19998.9     -3480.9  -90497.6
 61   20398.9     -3550.5  -94048.1
 62   20806.9     -3621.5  -97669.6
 63   21223.0     -3693.9 -101363.5
 64   21647.4     -3767.8 -105131.3

 65              +11040.2  -94091.1
 66              +11261.0  -82830.1
 67              +11486.2  -71343.9
 68              +11715.9  -59628.0
 69              +11950.3  -47677.7
 70              +12189.3  -35488.4
 71              +12433.1  -23055.4
 72              +12681.7  -10373.6
 73              +12935.4    2561.7
 74              +13194.1   15755.8
 75              +13457.9   29213.7
 76              +13727.1   42940.8
 77              +14001.6   56942.4
 78              +14281.7   71224.1
 79              +14567.3   85791.4
 80              +14858.7  100650.1
 81              +15155.8  115805.9
 82              +15458.9  131264.8
 83              +15768.1  147033.0
 84              +16083.5  163116.5

IRR = 0.0321
```

Simulation 4: $A^\star = 70$

```
per capita growth rate = 0.02
population growth = 0
U.S. survival curve
retirement at 70, worklife 45 yrs,
 retirement 15 yrs
retirement income: lambda = 0.5
workers per retiree = 4.583664368
implied tau = 0.109083

Age    Income  Retirement   Balance
 25   10000.0     -1090.8   -1090.8
 26   10200.0     -1112.6   -2203.5
 27   10404.0     -1134.9   -3338.4
 28   10612.1     -1157.6   -4496.0
 29   10824.3     -1180.7   -5676.7
 30   11040.8     -1204.4   -6881.1
 31   11261.6     -1228.5   -8109.5
 32   11486.9     -1253.0   -9362.6
 33   11716.6     -1278.1  -10640.6
 34   11950.9     -1303.6  -11944.3
 35   12189.9     -1329.7  -13274.0
 36   12433.7     -1356.3  -14630.3
 37   12682.4     -1383.4  -16013.8
 38   12936.1     -1411.1  -17424.9
 39   13194.8     -1439.3  -18864.2
 40   13458.7     -1468.1  -20332.3
 41   13727.9     -1497.5  -21829.8
 42   14002.4     -1527.4  -23357.2
 43   14282.5     -1558.0  -24915.2
 44   14568.1     -1589.1  -26504.3
 45   14859.5     -1620.9  -28125.2
 46   15156.7     -1653.3  -29778.6
 47   15459.8     -1686.4  -31465.0
 48   15769.0     -1720.1  -33185.1
 49   16084.4     -1754.5  -34939.6
 50   16406.1     -1789.6  -36729.2
 51   16734.2     -1825.4  -38554.7
 52   17068.9     -1861.9  -40416.6
 53   17410.2     -1899.2  -42315.7
 54   17758.4     -1937.1  -44252.9
 55   18113.6     -1975.9  -46228.8
 56   18475.9     -2015.4  -48244.2
 57   18845.4     -2055.7  -50299.9
 58   19222.3     -2096.8  -52396.7
 59   19606.8     -2138.8  -54535.5
 60   19998.9     -2181.5  -56717.0
 61   20398.9     -2225.2  -58942.2
 62   20806.9     -2269.7  -61211.9
 63   21223.0     -2315.1  -63526.9
 64   21647.4     -2361.4  -65888.3
 65   22080.4     -2408.6  -68296.9
 66   22522.0     -2456.8  -70753.7
 67   22972.4     -2505.9  -73259.6
 68   23431.9     -2556.0  -75815.6
 69   23900.5     -2607.1  -78422.7

 70              +12189.3  -66233.5
 71              +12433.1  -53800.4
 72              +12681.7  -41118.7
 73              +12935.4  -28183.4
 74              +13194.1  -14989.3
 75              +13457.9   -1531.4
 76              +13727.1   12195.7
 77              +14001.6   26197.4
 78              +14281.7   40479.1
 79              +14567.3   55046.4
 80              +14858.7   69905.0
 81              +15155.8   85060.8
 82              +15458.9  100519.8
 83              +15768.1  116287.9
 84              +16083.5  132371.4

IRR = 0.0340
```

C. Positive population growth plus flat survival curve

Simulation 5: $A^\star = 65$

```
per capita growth rate = 0.02
population growth = 0.0118
flat survival curve
retirement at 65, worklife 40 yrs,
 retirement 20 yrs
retirement income: lambda = 0.5
workers per retiree = 2.863199972
implied tau = 0.17463

Age    Income  Retirement   Balance
 25   10000.0     -1746.3   -1746.3
 26   10200.0     -1781.2   -3527.5
 27   10404.0     -1816.8   -5344.4
 28   10612.1     -1853.2   -7197.6
 29   10824.3     -1890.2   -9087.8
 30   11040.8     -1928.1  -11015.9
 31   11261.6     -1966.6  -12982.5
 32   11486.9     -2005.9  -14988.4
 33   11716.6     -2046.1  -17034.5
 34   11950.9     -2087.0  -19121.5
 35   12189.9     -2128.7  -21250.2
 36   12433.7     -2171.3  -23421.5
 37   12682.4     -2214.7  -25636.2
 38   12936.1     -2259.0  -27895.3
 39   13194.8     -2304.2  -30199.5
 40   13458.7     -2350.3  -32549.7
 41   13727.9     -2397.3  -34947.0
 42   14002.4     -2445.2  -37392.3
 43   14282.5     -2494.1  -39886.4
 44   14568.1     -2544.0  -42430.4
 45   14859.5     -2594.9  -45025.4
 46   15156.7     -2646.8  -47672.2
 47   15459.8     -2699.7  -50371.9
 48   15769.0     -2753.7  -53125.6
 49   16084.4     -2808.8  -55934.4
 50   16406.1     -2865.0  -58799.4
 51   16734.2     -2922.3  -61721.7
 52   17068.9     -2980.7  -64702.4
 53   17410.2     -3040.3  -67742.8
 54   17758.4     -3101.2  -70844.0
 55   18113.6     -3163.2  -74007.1
 56   18475.9     -3226.4  -77233.6
 57   18845.4     -3291.0  -80524.5
 58   19222.3     -3356.8  -83881.3
 59   19606.8     -3423.9  -87305.2
 60   19998.9     -3492.4  -90797.7
 61   20398.9     -3562.3  -94359.9
 62   20806.9     -3633.5  -97993.4
 63   21223.0     -3706.2 -101699.6
 64   21647.4     -3780.3 -105479.9

 65              +11040.2  -94439.7
 66              +11261.0  -83178.7
 67              +11486.2  -71692.4
 68              +11715.9  -59976.5
 69              +11950.3  -48026.2
 70              +12189.3  -35836.9
 71              +12433.1  -23403.9
 72              +12681.7  -10722.2
 73              +12935.4    2213.2
 74              +13194.1   15407.2
 75              +13457.9   28865.2
 76              +13727.1   42592.3
 77              +14001.6   56593.9
 78              +14281.7   70875.6
 79              +14567.3   85442.9
 80              +14858.7  100301.6
 81              +15155.8  115457.4
 82              +15458.9  130916.3
 83              +15768.1  146684.4
 84              +16083.5  162767.9

IRR = 0.0320
```

Simulation 6: $A^\star = 70$

```
per capita growth rate = 0.02
population growth = 0.0118
flat survival curve
retirement at 70, worklife 45 yrs,
 retirement 15 yrs
retirement income: lambda = 0.5
workers per retiree = 4.309554582
implied tau = 0.116021

Age    Income  Retirement   Balance
 25   10000.0     -1160.2   -1160.2
 26   10200.0     -1183.4   -2343.6
 27   10404.0     -1207.1   -3550.7
 28   10612.1     -1231.2   -4781.9
 29   10824.3     -1255.9   -6037.8
 30   11040.8     -1281.0   -7318.8
 31   11261.6     -1306.6   -8625.4
 32   11486.9     -1332.7   -9958.1
 33   11716.6     -1359.4  -11317.4
 34   11950.9     -1386.6  -12704.0
 35   12189.9     -1414.3  -14118.3
 36   12433.7     -1442.6  -15560.9
 37   12682.4     -1471.4  -17032.3
 38   12936.1     -1500.9  -18533.2
 39   13194.8     -1530.9  -20064.0
 40   13458.7     -1561.5  -21625.5
 41   13727.9     -1592.7  -23218.3
 42   14002.4     -1624.6  -24842.8
 43   14282.5     -1657.1  -26499.9
 44   14568.1     -1690.2  -28190.1
 45   14859.5     -1724.0  -29914.1
 46   15156.7     -1758.5  -31672.6
 47   15459.8     -1793.7  -33466.3
 48   15769.0     -1829.5  -35295.8
 49   16084.4     -1866.1  -37162.0
 50   16406.1     -1903.5  -39065.4
 51   16734.2     -1941.5  -41006.9
 52   17068.9     -1980.4  -42987.3
 53   17410.2     -2020.0  -45007.2
 54   17758.4     -2060.4  -47067.6
 55   18113.6     -2101.6  -49169.2
 56   18475.9     -2143.6  -51312.8
 57   18845.4     -2186.5  -53499.2
 58   19222.3     -2230.2  -55729.4
 59   19606.8     -2274.8  -58004.2
 60   19998.9     -2320.3  -60324.5
 61   20398.9     -2366.7  -62691.2
 62   20806.9     -2414.0  -65105.3
 63   21223.0     -2462.3  -67567.6
 64   21647.4     -2511.6  -70079.1
 65   22080.4     -2561.8  -72640.9
 66   22522.0     -2613.0  -75254.0
 67   22972.4     -2665.3  -77919.3
 68   23431.9     -2718.6  -80637.9
 69   23900.5     -2773.0  -83410.8

 70              +12189.3  -71221.6
 71              +12433.1  -58788.5
 72              +12681.7  -46106.8
 73              +12935.4  -33171.4
 74              +13194.1  -19977.4
 75              +13457.9   -6519.4
 76              +13727.1    7207.7
 77              +14001.6   21209.3
 78              +14281.7   35491.0
 79              +14567.3   50058.3
 80              +14858.7   64916.9
 81              +15155.8   80072.8
 82              +15458.9   95531.7
 83              +15768.1  111299.8
 84              +16083.5  127383.3

IRR = 0.0320
```

D. Positive population growth plus U.S. survival curve

Simulation 7: $A^\star = 65$

```
per capita growth rate = 0.02
population growth = 0.0118
U.S. survival curve
retirement at 65, worklife 40 yrs,
 retirement 20 yrs
retirement income: lambda = 0.5
workers per retiree = 4.080818789
implied tau = 0.122524

Age    Income  Retirement   Balance
 25   10000.0     -1225.2   -1225.2
 26   10200.0     -1249.7   -2475.0
 27   10404.0     -1274.7   -3749.7
 28   10612.1     -1300.2   -5050.0
 29   10824.3     -1326.2   -6376.2
 30   11040.8     -1352.8   -7729.0
 31   11261.6     -1379.8   -9108.8
 32   11486.9     -1407.4  -10516.2
 33   11716.6     -1435.6  -11951.8
 34   11950.9     -1464.3  -13416.1
 35   12189.9     -1493.6  -14909.6
 36   12433.7     -1523.4  -16433.1
 37   12682.4     -1553.9  -17987.0
 38   12936.1     -1585.0  -19572.0
 39   13194.8     -1616.7  -21188.7
 40   13458.7     -1649.0  -22837.7
 41   13727.9     -1682.0  -24519.7
 42   14002.4     -1715.6  -26235.3
 43   14282.5     -1750.0  -27985.3
 44   14568.1     -1784.9  -29770.2
 45   14859.5     -1820.6  -31590.9
 46   15156.7     -1857.1  -33447.9
 47   15459.8     -1894.2  -35342.1
 48   15769.0     -1932.1  -37274.2
 49   16084.4     -1970.7  -39244.9
 50   16406.1     -2010.1  -41255.1
 51   16734.2     -2050.3  -43305.4
 52   17068.9     -2091.4  -45396.8
 53   17410.2     -2133.2  -47530.0
 54   17758.4     -2175.8  -49705.8
 55   18113.6     -2219.4  -51925.2
 56   18475.9     -2263.7  -54188.9
 57   18845.4     -2309.0  -56497.9
 58   19222.3     -2355.2  -58853.1
 59   19606.8     -2402.3  -61255.4
 60   19998.9     -2450.4  -63705.8
 61   20398.9     -2499.4  -66205.2
 62   20806.9     -2549.3  -68754.5
 63   21223.0     -2600.3  -71354.8
 64   21647.4     -2652.3  -74007.2

 65             +11040.2  -62967.0
 66             +11261.0  -51706.0
 67             +11486.2  -40219.8
 68             +11715.9  -28503.8
 69             +11950.3  -16553.6
 70             +12189.3   -4364.3
 71             +12433.1    8068.8
 72             +12681.7   20750.5
 73             +12935.4   33685.8
 74             +13194.1   46879.9
 75             +13457.9   60337.8
 76             +13727.1   74064.9
 77             +14001.6   88066.6
 78             +14281.7  102348.3
 79             +14567.3  116915.6
 80             +14858.7  131774.2
 81             +15155.8  146930.0
 82             +15458.9  162389.0
 83             +15768.1  178157.1
 84             +16083.5  194240.6

IRR = 0.0436
```

Simulation 8: $A^\star = 70$

```
per capita growth rate = 0.02
population growth = 0.0118
U.S. survival curve
retirement at 70, worklife 45 yrs,
 retirement 15 yrs
retirement income: lambda = 0.5
workers per retiree = 6.581328251
implied tau = 0.0759725

Age    Income  Retirement   Balance
 25   10000.0      -759.7    -759.7
 26   10200.0      -774.9   -1534.6
 27   10404.0      -790.4   -2325.1
 28   10612.1      -806.2   -3131.3
 29   10824.3      -822.4   -3953.6
 30   11040.8      -838.8   -4792.4
 31   11261.6      -855.6   -5648.0
 32   11486.9      -872.7   -6520.7
 33   11716.6      -890.1   -7410.8
 34   11950.9      -907.9   -8318.8
 35   12189.9      -926.1   -9244.9
 36   12433.7      -944.6  -10189.5
 37   12682.4      -963.5  -11153.0
 38   12936.1      -982.8  -12135.8
 39   13194.8     -1002.4  -13138.2
 40   13458.7     -1022.5  -14160.7
 41   13727.9     -1042.9  -15203.7
 42   14002.4     -1063.8  -16267.5
 43   14282.5     -1085.1  -17352.5
 44   14568.1     -1106.8  -18459.3
 45   14859.5     -1128.9  -19588.2
 46   15156.7     -1151.5  -20739.7
 47   15459.8     -1174.5  -21914.2
 48   15769.0     -1198.0  -23112.3
 49   16084.4     -1222.0  -24334.2
 50   16406.1     -1246.4  -25580.6
 51   16734.2     -1271.3  -26852.0
 52   17068.9     -1296.8  -28148.7
 53   17410.2     -1322.7  -29471.4
 54   17758.4     -1349.2  -30820.6
 55   18113.6     -1376.1  -32196.7
 56   18475.9     -1403.7  -33600.4
 57   18845.4     -1431.7  -35032.1
 58   19222.3     -1460.4  -36492.5
 59   19606.8     -1489.6  -37982.1
 60   19998.9     -1519.4  -39501.4
 61   20398.9     -1549.8  -41051.2
 62   20806.9     -1580.7  -42631.9
 63   21223.0     -1612.4  -44244.3
 64   21647.4     -1644.6  -45888.9
 65   22080.4     -1677.5  -47566.4
 66   22522.0     -1711.1  -49277.5
 67   22972.4     -1745.3  -51022.7
 68   23431.9     -1780.2  -52802.9
 69   23900.5     -1815.8  -54618.7

 70             +12189.3  -42429.4
 71             +12433.1  -29996.4
 72             +12681.7  -17314.6
 73             +12935.4   -4379.3
 74             +13194.1    8814.8
 75             +13457.9   22272.7
 76             +13727.1   35999.8
 77             +14001.6   50001.4
 78             +14281.7   64283.1
 79             +14567.3   78850.4
 80             +14858.7   93709.1
 81             +15155.8  108864.9
 82             +15458.9  124323.8
 83             +15768.1  140092.0
 84             +16083.5  156175.5

IRR = 0.0455
```